1 第10教学周11.7

7.2 Regularity and approximation theorems

X: Locally compact Hausdorff space.

命题1.1 (Regularity). *Every Radon measure is inner regular on all of its σ-finite sets.*

推论1.2. *Every σ-finite Radon measure is regular. If X is σ-compact, every Radon measure on X is regular.*

命题1.3 (Density). *If μ is a Radon measure on X, $C_c(X)$ is dense in $L^p(X)$ for $1 \le p < \infty$.*

定理1.4 (Lusin). *Suppose that μ is a Radon measure on X and $f : X \to \mathbb{C}$ is a measurable function that vanishes outside a set of finite measure. Then for any $\epsilon > 0$ there exists $\phi \in C_c(X)$ such that $\phi = f$ except on a set of measure $< \epsilon$. If f is bounded, ϕ can be taken to satisfy $\|\phi\|_u \le \|f\|_u$.*

定理1.5 (Tietze extension). *Let $K \subset X$ be compact. If $f \in C(K)$, there exists $g \in C_c(X)$ such that $g\big|_K = f$.*

7.3 The dual of $C_0(X)$

X: Locally compact Hausdorff space.

$C_0(X)$ is the uniform closure of $C_c(X)$.

命题1.6. $C_0(X) = \{f \in C(X) : f$ *vanishes at inifinity*$\}$.

引理1.7. *If $f \in [C_0(X)]^*$, there exist positive functional $I^{\pm} \in [C_0(X)]^*$ such that*

$$I = I^+ - I^-. \tag{1.1}$$

定理1.8 (Riesz Representation Theorem on $C_0(X)$). *Let X be an LCH space, and for $\mu \in M_r(X)$ and $f \in C_0(X)$ let $I_\mu(f) = \int f d\mu$. Then the map $\mu \mapsto I_\mu$ is an isometric isomorphism from $M_r(X)$ to $[C_0(X)]^*$.*

推论1.9. *If X is a compact Hausdorff space, then $[C(X)]^*$ is isometrically isomorphic to $M_r(X)$.*

7.4 convergence of measures

X: Locally compact Hausdorff space.

$$C_c(X) \subset C_0(X) \subset C_b(X). \tag{1.2}$$

- vague convergence:

$$\int f d\mu_n \to \int f d\mu_0, \quad \forall f \in C_c(X). \tag{1.3}$$

- weak convergence:

$$\int f d\mu_n \to \int f d\mu_0, \quad \forall f \in C_0(X). \tag{1.4}$$

- narrow convergence:

$$\int f d\mu_n \to \int f d\mu_0, \quad \forall f \in C_b(X). \tag{1.5}$$

作业

P_{215} **2;** P_{220} **7,8,9;**